Einstein 1916:
Final Mathematical and Reproducibility Audit

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8 November 2026
Publication identifier: 10.5281/zenodo.24064553

Abstract

This report records the cumulative audit of the six research checkpoints underlying *The Foundations of General Relativity: A Source–Readout Reconstruction of Einstein's 1916 Theory*. It distinguishes mathematical proof, source-text fidelity, implementation tests and byte-level reproducibility. The final manuscript contains the complete conditional reconstruction and essential proofs. The six retained suites reproduce all 530 exact check groups and eight numerical diagnostics. A separately written publication audit adds 40 exact groups, including 11 controls against invalid alternatives. The report inventories assumptions, theorem status, source corrections, comparison with established formulations, retained limitations and the public release's reproducibility contract. It is an internal AI-assisted mathematical audit, not independent peer review or proof-assistant certification.

Funding: No external funding.
Record status: The DOI was supplied as reserved. Public release of its repository record is not asserted.

1 Scope and audit disposition

The task is a standalone reconstruction of Einstein's 1916 article, including Parts A–E and numbered equations (1)–(75), not a new gravitational theory or a quantum-gravity completion. The manuscript supplies 101 correspondence entries: all 75 principal equation numbers and 26 lettered variants. Its source architecture, coframe realization, derivative and variation maps, conservation identities, constitutive matter construction, controlled approximations and observational dependency completion are integrated into one mathematical article.

The strongest established claim is conditional local operation and solution-set recovery under explicit physical and mathematical hypotheses. Native admission supplies an architectural antecedent; it does not select the continuum, Lorentz signature, Einstein action, matter laws or constants. The standard coframe and connection machinery is acknowledged rather than presented as a new invention. The complete manuscript and public code make the relevant distinction inspectable.

The audit disposition is: the requested derivation record has been consolidated, the stated calculations have been checked and repaired where necessary in the preceding record, and no additional algebraic change was required by the final 40-group publication suite. This is a positive

internal release result, not a guarantee against undiscovered errors or journal acceptance. Claims that require a different physical theory, global topology, arbitrary boundary-charge equivalence, general material well-posedness or new astronomical data remain outside the theorem rather than being represented as completed results.

2 Inputs, integrity and scientific continuity

The authoritative original is Einstein's German paper, printed pp. 769–822 [5]. The uploaded PDF and its page images were consulted, with the publisher-hosted PDF used for comparison. The 1915 perihelion article and Schwarzschild's mathematical solution were inspected only for the stated external dependency [4, 10]. The public source-architecture definitions are stated sufficiently for the one initial witness used in the manuscript [9].

Each uploaded research archive was extracted, every entry in its recorded payload manifest checked, and every nested predecessor archive compared byte for byte with the separately uploaded predecessor. The full hashes and member checks are in `provenance/input_integrity.json`.

Checkpoint	Manifest entries	Exact groups	Outcome
I01: metric and measurement	12	54	All pass
I02: differential calculus	18	94	All pass
I03: vacuum/action/energy	20	76	All pass
I04: matter/conservation	26	94	All pass
I05: fluid/electromagnetic	31	111	All pass
I06: Newtonian/observational	31	101	All pass

The six complete scientific result objects, not merely their pass counts, match the retained objects. The eight numerical diagnostic records also match. The final Python code for each retained stage suite is unchanged. Its source hashes are listed in the script-provenance register. Historical private instructions, full nested research archives and third-party scans are not copied into the public release.

3 Mathematical audit by dependency chain

3.1 Admission, metric and tensor operations

The native two-vertex support is admitted through its membership in the specified initial witness class. It carries no physical time, metric or desired solution. All physical data enter at the realization layer. The local metric section was checked constructively: when the fourth coordinate direction is timelike, the Schur-complement matrix is positive definite and its Cholesky factor gives an explicit calibration. Equal metric readouts imply exactly a local Lorentz relation. The proof covers all local target metrics, not only test examples.

The tangent lift covers every symmetric metric variation. The joint stress tangent lift accounts for variation of the calibration as well as the calibrated stress; holding calibrated stress fixed is not confused with holding coordinate stress fixed. Tensor products and contractions are proved by slot cancellation, including arbitrary finite sums. The source quotient is not made coarser than the outputs needed later: uncalibrated null directions do not determine clock scale, and electromagnetic stress does not determine a primitive electromagnetic field.

3.2 Differentiation, curvature and local scope

The comparison connection is constructed from coframe structure coefficients by a unique solution of the metric-compatibility and zero-torsion equations. Raw defects are retained before restriction. The coordinate connection is a readout, and its raw nonmetricity and torsion identities are explicit. Vector and covector derivative intertwiners extend to all tensor types, including the appended derivative slot.

Curvature recovery follows from an operator commutator, retaining the ordered quadratic product. The original contraction sign and Einstein's negative Christoffel convention were checked against the source. The local flatness converse uses a sufficiently small simply connected neighborhood; it is not promoted to global constant coordinates or trivial holonomy. A coordinate volume choice is not an internal Lorentz transformation. The connection variation includes the dependent response of the comparison connection.

3.3 Variation, field equations and gravitational energy

The source first-derivative action equals the declared chart-tagged extension of Einstein's Hamiltonian function. Its exact variation includes the boundary term and all symmetric inverse-metric directions. The key operation order is unrestricted variation followed by evaluation in a unit-volume chart. Constraining all variations to that slice would lose the trace equation; an explicit nonzero-Ricci example passes the resulting weaker equation.

The separately derived curvature-action comparison retains both its boundary vector and the difference of unrestricted derivatives transverse to the volume slice. A fixed-metric boundary variation with nonzero curvature-action response demonstrates why boundary ensembles cannot be silently identified. The canonical energy current is recovered from the source action through the chain rule, and its off-shell divergence identity is proved. A flat unit-volume coordinate transformation produces a nonzero current from a zero Cartesian one, so the non-tensor status is preserved.

3.4 Matter coupling and conservation

Trace reversal in four dimensions is an involution, not a trace-free projection. The same-realization addition of matter stress to gravitational energy recovers the complete field equation, including the trace. The original double-differentiation proof is supplied with its unrestricted volume defect. Matter and gravitational exchange have opposite signs, tested on a coupled nonzero-exchange fixture. Covariant matter conservation also follows through a separately implemented curvature identity. Fixed-region balance retains actual surface fluxes; conservation is not used as a substitute for the full gravitational equation or arbitrary constitutive closure.

3.5 Primitive matter and observable operations

The fluid calculation preserves Einstein's density convention: comoving energy is $\rho - p$. Its scalar and transverse equations reconstruct the complete divergence on normalized timelike velocities. Off-normalization defects and the $\rho = 0$ velocity degeneracy are explicit. The electromagnetic construction preserves the original potential sign, metric-dependent index raising and second-index divergence. The stress transfer identity retains both Maxwell residuals. The source's zero-transfer branch is not promoted to a general equation of charge motion.

The Newtonian calculation keeps weak amplitude, derivative control, slow motion, quasi-static variation and pressure assumptions distinct. A Green-function coefficient and boundary join fix the mass parameter. The static spherical completion is labelled as additional dependency work rather than attributed to the first-order metric in the 1916 article. Optical variation is performed before fixing direction; a finite endpoint term is retained. The finite output is a coordinate-direction change until local observer frames are specified. The orbital turning-point integral is regularized before expansion, and its positive-domain and remainder bounds are stated explicitly.

4 Theorem-status register

The following results have proofs in the manuscript. The labels, hypotheses and evidence appear in machine-readable form in `registers/theorem_status.csv`. "Proof in manuscript" means an explicit mathematical argument in this work, not formal proof-assistant certification or independent referee approval.

Result	Claim and evidence	Hypotheses and limitation
2.1 (p. 5)	Composition and information-loss criterion. Elementary proof in text.	Finite dependency graph; equality on actual domains; fibre congruence for future operations. Set-level factorization is not a regularity theorem.
3.1 (p. 6)	Local metric section and complete fibre. Constructive Cholesky section and fibre proof.	A02-A03; fixed coordinates; nondegenerate Lorentz metric. No unique native antecedent or global frame.
4.1 (p. 8)	Tensor algebra recovery. Slot cancellation proof.	Invertible E; arbitrary finite rank and tensor sums. No restriction to decomposable tensors.
4.2 (p. 9)	Compatible comparison existence and uniqueness. Explicit structure-coefficient formula and uniqueness proof.	A02,A04; smooth invertible E. Physical compatibility is selected at realization layer.
4.3 (p. 10)	Derivative operation recovery. Vector/covector proof plus products and contractions.	Raw omega and invertible E; compatible sector for Einstein calculus. Appended derivative slot is included.
6.1 (p. 12)	Full curvature recovery. Operator commutator proof.	Raw comparison domain. Einstein short covector form also needs zero torsion.
7.1 (p. 15)	Field action and variation recovery. Exact variation, boundary expression and tangent surjectivity.	A04,A06; chart-tagged H; all symmetric variations. Not off-gauge covariant action equivalence.
10.1 (p. 19)	Original conservation identity with volume defect. Antisymmetrization and trace calculation.	Smooth compatible geometry; third derivatives. Zero only after stated volume specialization.
11.1 (p. 23)	Reversible energy/Euler split. Product rule and orthogonal projection proof.	A09 normalized timelike velocity. No arbitrary EOS well-posedness or division by rho.
12.1 (p. 24)	Local potential section and gauge fibre. Explicit homotopy integral.	Smooth closed two-form on star-shaped neighborhood. No global exactness or arbitrary boundary quotient.
12.2 (p. 26)	Complete electromagnetic transfer residual. Product and cyclic identity proof.	Compatible calculus; arbitrary antisymmetric field/current. Zero-transfer conservation is conditional.
13.1 (p. 28)	Finite-jet operation recovery. Taylor, product and inverse identities.	A12; enough derivatives and bounded inverse. Not long-time uniform Newtonian convergence.

Result	Claim and evidence	Hypotheses and limitation
16.1 (p. 34)	Boundary-sensitive optical comparison. Direction-dependent Euler-Lagrange calculation.	A13-A14; fixed ray chart; first-order variation. Finite coordinate direction change, not observer-invariant angle.
17.1 (p. 36)	Perihelion coefficient and error bound. Root factorization and Taylor bound.	A13,A15; stated regular bound branch. Leading Kepler conversion; no fitted-element error bound.
18.1 (p. 37)	Cumulative conditional reconstruction. Composition of displayed operation and reverse-map proofs.	A01-A16 as applicable to each target. No native law selection, new dynamics or independent peer review.

5 Source fidelity and substantive correction decisions

Six printed-source issues are isolated. These are consistency decisions for the supplied copy, not claims concerning the origin or novelty of an erratum.

5.1 Rank terminology, p. 784

The copy uses the word for symmetric tensors in a sentence denying tensors above rank four, after a discussion of completely antisymmetric tensors. A fifth tensor power of a nonzero covector disproves the symmetric reading. The manuscript retains the correct exterior-algebra cutoff and leaves the arbitrary-rank symmetric transformation algebra intact.

5.2 Exceptional trace parameter, p. 804

The footnote equation $B+\lambda g\,\mathrm{tr}_g\,B=0$ implies $B=0$ only for $\lambda\neq -1/4$. Its trace is $(1+4\lambda)\,\mathrm{tr}_g\,B=0$. The explicit calibration with $\mathrm{Ric}=-g/3$ supplies a nonzero counterexample at the exceptional parameter. The selected vacuum equation is not changed.

5.3 Action derivative sign, p. 805

The unnumbered line before (48) disagrees with numbered (48). Direct differentiation gives a positive coefficient of $\Gamma_{\mathrm{E}}\partial(\delta q)$, agreeing with (48). Both the general proof and independent final lapse variation test reject the negative coefficient. The discrepancy is not justified through a determinant-constrained variation.

5.4 Gravitational exchange sign, p. 809

The unnumbered gravitational relation must have divergence minus exchange, while numbered (57) for matter has divergence plus exchange. The coupled dust example has nonzero exchange and makes the printed gravitational residual nonzero. Numbered (57), (57a), total conservation and the full field equation remain mutually consistent with the documented correction.

5.5 Mass normalization and propagated bending coefficient, pp. 819 and 822

The potential from (68a) and $g_{44} = 1 - \alpha/r$ require $\alpha = \kappa M/(4\pi)$, not the printed (70a) denominator 8π. The parameter bending $2\alpha/b$ remains correct; the printed mass expression in (74) propagates the earlier half-normalization. The reconstruction keeps the same mass, units, Green flux and asymptotic clock across the join.

5.6 Light-speed expansion, p. 822

The positive printed correction to the directional coordinate speed fails the preceding null equation at first order. The negative expansion is fixed by that equation and gives the stated inward-positive asymptotic bending. The finite endpoint comparison is an additional qualification of an extended optical operation, not a correction to the source's infinite-endpoint result.

5.7 Qualified prose and dependency boundaries

The zero-transfer assertion for free charges is used only on its explicit algebraic branch; Maxwell's equations do not supply a missing charge-motion law. Equation-count statements are not general well-posedness theorems. The final perihelion expression is recovered through the cited 1915 cubic and the labelled radial completion. Historical astronomical numbers remain source-reported rather than independently recomputed. These qualifications are distinct from typographical corrections.

6 Final publication audit and preserved development failures

The separately written publication checker completed 40 exact groups, all passing, including 11 expected-failure controls. It uses a dense non-diagonal calibration and stress for inverse and variation maps, direct differentiation of a lapse-dependent metric action, an exact finite-boundary integral, nonzero coupled exchange, component electromagnetic energy, optical primitives and boundary terms, source mass normalization, orbit root factorization and derivative-topology controls. The tests are authored within the same workflow; they are not an independent external implementation by a different investigator.

Two initial Part D negative controls failed during development. The actual retained result object reports 103 of 105 groups passing. One incorrectly expected the raw algebraic fluid decomposition to fail without normalization; only the projection and orthogonality interpretation fails. The other selected a boosted constant potential for which the bare antisymmetric derivative happened to vanish. The corrected final suite tests the actual failing claims and uses a constant coordinate $\mathrm{d}x^1$ potential. Initial code, result and log are retained as historical evidence in `audit/development/`; those files are not counted as current passing executions.

An earlier Part C-to-matter boundary calculation timed out in a general symbolic integration request. It was replaced by an explicit antiderivative whose derivative and endpoints are checked exactly. An incomplete execution is not a pass. The final suites avoid that stalled operation and reproduce the mathematical finite-flux result.

No new discrepancy requiring alteration of the six final stage algorithms was found by the consolidation. The manuscript sharpens finite-endpoint measurement terminology, gives the bound-orbit energy factorization and explicitly displays the radial comparison connection. The forty

publication groups are supplementary regressions against these final conventions, not an assertion that a finite test suite proves the entire theory.

7 Novelty assessment and prior formulations

Coframes, compatible connections, differential forms and metric/frame equivalence are established mathematics. Peldan's review surveys gravitational actions [8]; Dadhich and Pons treat metric/Palatini equivalence for an independently varied connection [2]. The present dependent-connection reconstruction does not supersede those results or claim an independent-connection variation that was not performed.

Boundary equivalence is also an established research subject. The tetrad symplectic-potential and boundary analyses of De Paoli–Speziale and Oliveri–Speziale, and the metric-versus-tetrad covariant-phase-space work of Barbero G. and collaborators, demonstrate why boundary contracts must be specified [1, 3, 7]. Iyer–Wald provides relevant Noether-charge context [6]. The present paper does not establish equivalence of all symplectic potentials, corner data or charges.

The defensible contribution is the integrated source–readout organization of the full historical chain, explicit primitive reverse sections and variation coverage, complete residuals and same-realization joins, information-loss counterexamples, boundary and approximation contracts, and a public equation correspondence and verification release. This is more than a final-equation renaming, but it is not a claim that each identity is historically new. The comparison was targeted to primary literature relevant to these claims; it was not an exhaustive priority search. Journal suitability and external novelty judgment remain editorial and peer-review questions.

8 Reproducibility contract and release checks

The retained exact suites contain 530 groups, including 88 expected-failure controls. The publication audit adds 40 groups, including 11 controls. Thus the final executed exact total is 570 groups with zero failures; the tests are not statistically independent experiments. Eight separate numerical diagnostics use mpmath at 65 decimal digits and check strict inequalities between measured integration error and the proved analytic bounds. They are not interval-certified error estimates or empirical observations.

The tested environment is Python 3.13.5, SymPy 1.14.0 and mpmath 1.3.0. The package command

```
python verification/run_all.py -output replay -jobs 2
```

executes every suite, stores fresh logs and outputs, and compares complete result objects and scientific content separately. Dependency installation and exact reproduction instructions are in the package README. The clean-extraction replay receipt records the actual final release test; no future or unexecuted result is represented as complete.

The LaTeX source is editable and compiles with pdfLaTeX. The build script detects available BibTeX variants and includes the generated bibliography as a fallback. The final manuscript and audit PDFs are rendered and visually inspected; final build logs and a layout receipt are included. Every archive is opened and checked, and the release manifest binds all public payload

files. Manifests do not include themselves or their own binding receipts, avoiding a circular hash definition.

9 Limitations, privacy and publication status

The result is a conditional classical reconstruction. It does not prove native source-law selection, a new physical theory, elimination of nonlinear curvature, quantum consistency, general global recovery, arbitrary material initial-value uniqueness or a new astronomical measurement. Smoothness, nondegeneracy, local domains and each boundary/approximation protocol are necessary parts of the result. All essential proofs are in the manuscript; private working protocols are not required to reproduce the public checks.

The public release contains no private protocol text or prompts, no third-party scans, no font files, and no account credentials. Original sources are cited and their inspected roles recorded. The prior private research archives are represented by provenance hashes and selected authored scientific code, not copied wholesale. Repository licensing and any journal-specific declarations are for the author to set when depositing. No repository upload, DOI publication or journal submission has been performed by this audit.

10 Conclusion

The reconstruction has been consolidated into a complete scholarly manuscript with explicit hypotheses, source qualifications, proof-bearing operations, numerical-domain restrictions and a reproducible public package. All executed final tests pass and the retained result objects reproduce. The final audit supports release of the work as the stated conditional reconstruction, while preserving the distinction between internal verification and independent mathematical review.

References

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